Bending Moments

The bending moment (M) is a primary design concern for beams. It can be used to determine longitudinal stress and deflection. It immediately shows the required designer the required performance for every beam section.

External load can be divided into several types:

- concentrated or point loads, measured in Newtons and represented by W
- uniformly distributed loads, measured in Newtons per metre and represented by w
- moment or rotating loads, measured in Newton-metres and represented by M_x

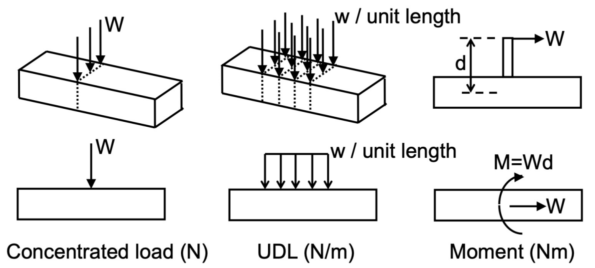


Figure 1: Types of external load, taken from Week 1 Overheads, page 2

There are three main types of beam support used:

- pin support (constrained in horizontally and vertically)
- roller support (constrained horizontally and downwards)
- cantilever (fixed) support (fully constrained)

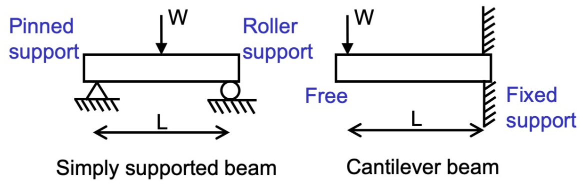


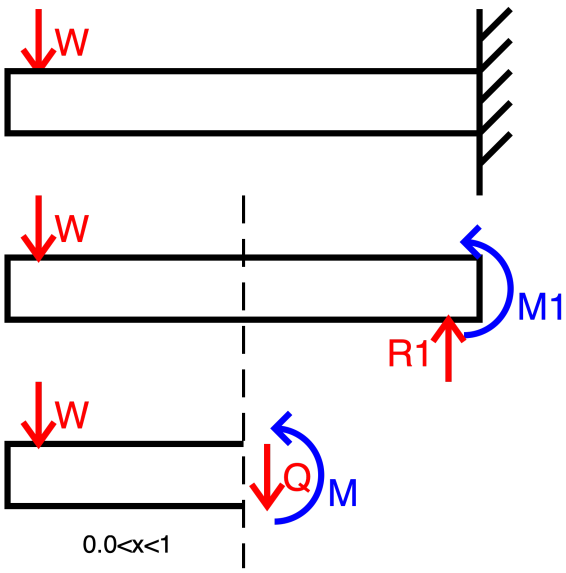
Figure 2: Types of support, taken from Week 1 Overheads, page 3

Method of Sections

The Method of Sections allows you to easily visualise how the vertical forces and bending moments vary along the length of a beam. By convention, the bending moment M is rotating counterclockwise, and the shear force Q acts downwards (so a negative value for each is clockwise and upwards, respectively.)

To perform the Method of Sections on a beam:

1. Split the beam between each force applied to it.
2. Starting from the left-hand side, work along. For each section:
   (a) Draw a free-body diagram of everything from the left-hand side of the beam to the end of the section (i.e. encompassing all previous sections)
   (b) Find the equilibrium of moments by considering the right-most point a ”pivot” point.
   (c) Find the equilibrium of vertical forces.
3. Graph the moments and vertical forces along the length of the beam.



Worked Example 1

1. Generate a free-body diagram for the whole beam.

   Use this to find equilibria for the whole beam.

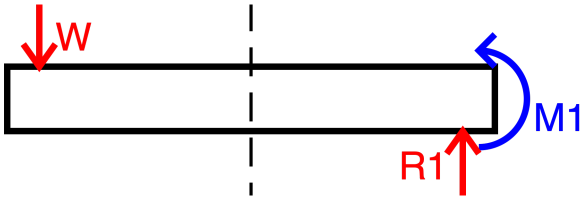
   - moments at RH end:

   $$M_1 + WL = 0$$
   $$\therefore M_1 = -WL$$

   The cantilever support exerts a moment M_1 on the beam. Since a moment can be defined as a force at a distance, it is balanced by $W \cdot L$ where L is the length of the beam.

   - vertical forces:

   $$W - R_1 = 0$$
   $$\therefore W = R_1$$



2. Slice the beam into sections between each force. In this case, since we only have W and R_1, we only need one section.

   We'll say we make the cut at some distance x between the two forces, such that the equations of equilibrium we produce will be accurate for any position x which satisfies the requirements $0 < x < 1$.

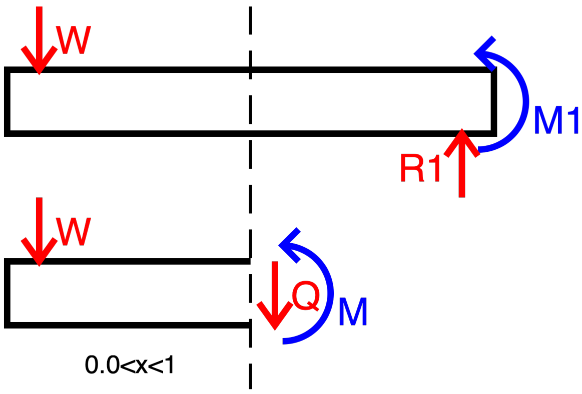
   Repeat the process of finding the equilibria - but this time, add in the internal shear force Q and the internal moment M at the cut end.

   - moments:

   $$M + Wx = 0$$
   $$\therefore M = -Wx$$

   - forces vertically:

   $$W + Q = 0$$
   $$\therefore Q = -W$$



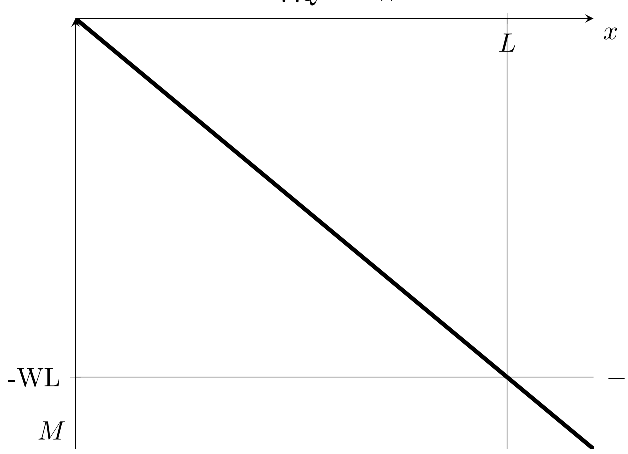


Figure 3: Graph of moments along the beam length

Figure 4: Graph of vertical forces along the beam

Worked Example 4

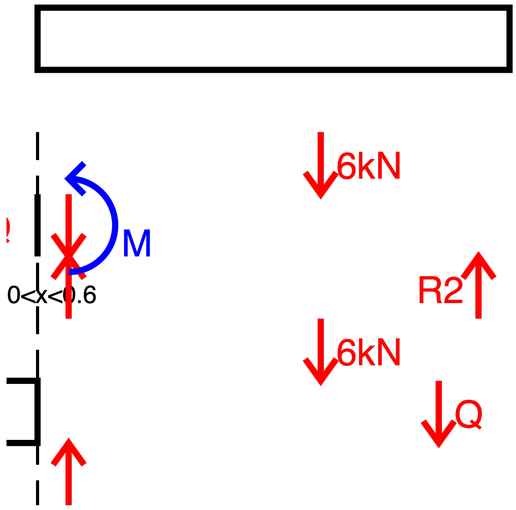
1. Generate an FBD for the whole beam and find equilibria.

   - moments:

$$
\begin{aligned}
&0.4m \cdot 6kN - 2m \cdot R_1 = 0\\
\therefore &2.4kN = 2m \cdot R_1\\
\therefore &R_1 = 1.2kN
\end{aligned}
$$

   - vertical forces:

$$
\begin{aligned}
6kN =& R_1 + R_2\\
=& 1,2kN + R_2\\
\therefore R_2 =& 4.8kN
\end{aligned}
$$



2. Take sections of the beam and calculate moments and vertical forces for each.

   -