Bending Stress

The bending equation is:

$$\frac{M}{I} = \frac{\sigma_x}{y} = \frac{E}{R}$$

where

- **M** is the bending moment, in Newton-metres
- **I** is the second moment of area, in metres to the fourth power
- σ_x is longitudinal stress, in Newtons per square metre
- **y** is the distance of the furthest point of the beam from the neutral axis, in metres
- **E** is Young's modulus, in Newtons per square metre
- **R** is the radius of curvature of the neutral axis, in metres.

It is used to relate the *longitudinal stress* to the *bending moment* to ensure that the maximum bending moment doesn't cause the longitudinal stress to exceed safe values. Or, more simply, to calculate if the beam is strong enough.

Some assumptions are made when the bending equation is used. These are:

- the beam is only subjected to a bending moment. There are no point loads applied.
- planar cross-sections remain planar during the bending - i.e. the material curves in an arc.
- the beam deforms elastically, and is made of a single material.
- there is a vertical axis of symmetry in the cross-section of the beam.

The Neutral Axis and the Centroid

The *neutral axis* x is the axis which passes through the centroid of the beam for each cross-section. The centroid is the centre of mass for a cross-section of a beam.

The Second Moment of Area

The *second moment of area* I (sometimes I_Z) is given by two equations, one for a rectangle and the other for a circle.

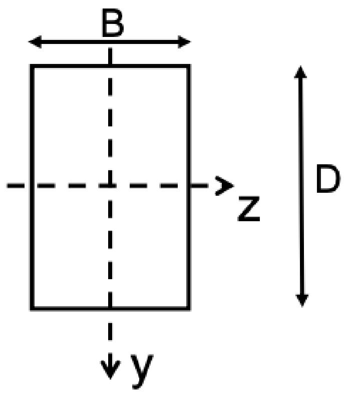

Figure 1: The second moment of area for a rectangle is given by $I = \frac{BD^3}{12}$

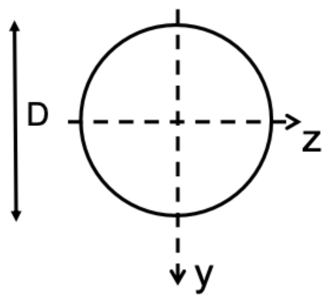

Figure 2: The second moment of area for a circle is given by $I = \frac{\pi D^4}{64}$

Values for I can be added and subtracted to form more complex shapes.

For instance, a hollow box beam can be conceptualised as one rectangle from which another is subtracted.

However, in the case of an I-beam, this becomes more complicated. It can be conceptualised as a rectangle from which two others (the sides) are subtracted, or it can be conceptualised as three rectangles added together. In the latter case, we need to use the *parallel axis theorem.*

Parallel Axis Theorem

The parallel axis theorem is used when the neutral axis of a section of beam doesn't intersect the centroid. This is common in the case of the I-beam, where the top and bottom rectangles are offset by some distance d from the neutral axis od the beam as a whole. In this case, we can calculate the second moment of inertia with

$$I = \overline{I} + Ad^2$$

where

- $\overline{I}$ is the second moment of inertia of the shape
- A is the area of the shape, and
- d is the distance from the neutral axis of the shape to the neutral axis of the beam as a whole

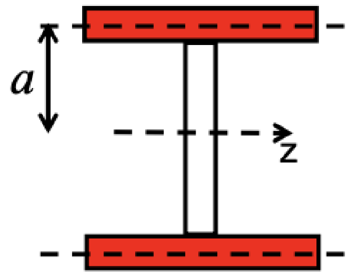

Figure 3: Here, the segments which will require the parallel axis theorem are shown in red.