Applications of Linear and Logarithmic Functions

Point-Slope Formula

The straight line formula $y = mx + b$ is perfectly good, but in many cases, the point-slope equation $y - b = m(x - a)$ (for some point (a, b) is more efficient. Given a slope of $m = 2$ and a point $(1, 3)$:

1. Substitute in values.
2. Rearrange to solve for y.
3. Multiply out.
4. Simplify.

$$\begin{aligned} y - b &= m(x - a) \\ y - 3 &= 2(x - 1) && (1) \\ y &= 2(x - 1) + 3 && (2) \\ &= 2x - 2 + 3 && (3) \\ &= 2x + 1 && (4) \end{aligned}$$

Given two points (x, y) and (a, b) on a slope, we can say that $m = \frac{y-b}{x-a}$, or $m = \frac{\Delta y}{\Delta x}$

Finding Linear Correlations

Experimental data, believed to be related by $R = mT + c$, are shown in the table Data of T and R. Plotting these points shows that they approximately lie on a straight line.

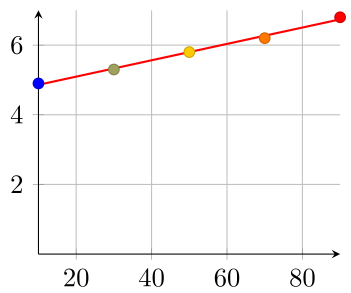
Table 1: Data of T and R

T	10	30	50	70	90
R	4.9	5.2	5.8	6.2	6.8

Adding a line of best fit (shown in Plot of T and R) and taking the points $(30, 5.25)$ and $(70, 6.25)$, we can find the gradient: $m = \frac{y-b}{x-a} = \frac{1}{40} = 0.025$.

Using one of these points, we can then use one of these points to get the equation of the line: $R - b = m(T - a) \therefore R = 0.025T + 4.5$.

Figure 1: Plot of T and R



Finding Logarithmic Correlations

Values may not be linearly related; data can be manipulated by plotting other options. The equation $y = mt^2 + c$ has no good straight line solution. Instead, we can define $x = t^2$ and produce $y = mx + c$.

Table 2: Data of t, y and x

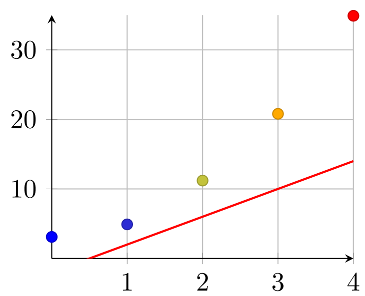
t	0	1	2	3	4
y	3.1	4.9	11.2	20.8	34.9
x	0	1	4	9	16

By plotting x against t (shown in Plot of t, y and x) and taking the points $(0, 3)$ and $(10, 23)$, we can find the gradient: $m = \frac{y-b}{x-a} = \frac{20}{10} = 2$.

Using one of these points, we can then use one of these points to get the equation of the line: $y - 3 = 2(x - 0) \therefore y = 2x + 3$. Substituting t^2 back in will produce $y = 2t^2 + 3$.

The process of manipulating variables so that they are related by a single straight line is called *linearisation*.

Figure 2: Plot of t, y and x



Log-Log and Log-Linear Laws

In the real world, variables will often be related by the *power law* or the *exponential law*. Logarithmic linearisation of these data allow for better understanding of them.

Given the relationship $y = at^b$ where y and t are some variables of interest, this relationship can be linearised by taking the logarithm of both sides and then applying the laws of logs. This will produce a straight line, and so $\ln(y)$ can be plotted against $\ln(t)$. The resulting graph is called *log-log*, since both axes are logarithms.

$$\begin{aligned} y &= at^b \\ \ln(y) &= \ln(at^b) \\ &= \ln(a) + \ln(t^b) \\ &= \ln(a) + b \cdot \ln(t) \\ w &= \ln(y) \\ x &= \ln(t) \\ \therefore w &= \ln(a) + bx \end{aligned}$$

Exponential Relationships

Exponential relationships can also be linearised. For instance, $y = Ab^t$ (where, again, y and t are the relevant variables) can be linearised as shown.

The resulting equation $w = \ln(A) + t\ln(b)$ is a straight line, and therefore can be plotted with y against t. This plot is called *log-linear*, since one axis is logarithmic and the other linear.

$$\begin{aligned} y &= Ab^t \\ \ln(y) &= \ln(Ab^t) \\ &= \ln(A) + \ln(b^t) \\ &= \ln(A) + t\ln(b) \end{aligned}$$

For a question asking if data are exponential, such as those shown in Data of decay rates, the following method can be used:

1. Express the relationship as an exponential. In this case, $A(t) = A_0e^{-kt}$ where k is some constant and A_0 is activity when $t = 0$.
2. Linearise the equation. $\ln A(t) = \ln(A_0e^{-kt}) = \ln(A_0) + \ln(e^{-kt}) = \ln(A_0) - kt$.
3. Plot $ln(A)$ against t and find a straight line.

Table 3: Data of decay rates

Time (s)	Activity
0	1000
5	860
10	741
15	637
20	549
25	472
30	405
35	350
40	301
45	260
50	225
55	190