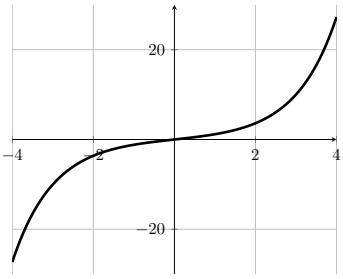
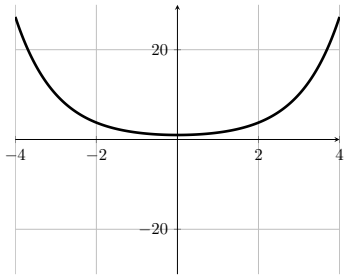
Hyperbolic Functions

Hyperbolic Trigonometry

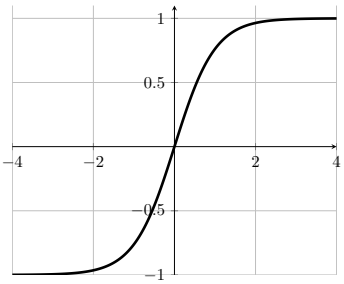
The hyperbolic trigonometric functions are defined as $sinh(x) = \frac{1}{2}(e^x - e^{-x})$, $cosh(x) = \frac{1}{2}(e^x + e^{-x})$ and $tanh = \frac{sinh(x)}{cosh(x)}$.



Graph of $sinh(x)$



Graph of $cosh(x)$



Graph of $tanh(x)$

Given some expression $ae^x + be^{-x}$, it can always be expressed in terms of $\sinh(x)$ and $\cosh(x)$:

1. Any ae^x can be found with $a\cosh(x) + a\sinh(x)$.
2. Any be^{-x} can be found with $b\cosh(x) - b\sinh(x)$.
3. Therefore, $ae^x + be^{-x}$ can always be found with $(a+b)\cosh(x) + (a-b)\sinh(x)$.

Examples

Express $5e^{4x} - 7e^{-4x}$ in terms of $\cosh(4x)$ and $\sinh(4x)$.

$$\begin{aligned}
\cosh(x) &= \frac{1}{2}(e^x + e^{-x}) \\
\therefore \cosh(4x) &= \frac{1}{2}(e^{4x} + e^{-4x}) \\
\sinh(x) &= \frac{1}{2}(e^x - e^{-x}) \\
\therefore \sinh &= \frac{1}{2}(e^{4x} - e^{-4x})
\end{aligned}$$

$$\cosh(4x) + \sinh(4x) = \frac{1}{2}(e^{4x} + e^{-4x}) + \frac{1}{2}(e^{4x} - e^{-4x}) = e^{4x}$$

Express $9e^x + 2e^{-x}$ in terms of $\cosh(x)$ and $\sinh(x)$.

$$\begin{aligned}
\cosh(x) &= e^x + e^{-x} \\
\sinh(x) &= e^x - e^{-x} \\
9e^x &= 9\cosh(x) + 9\sinh(x) \\
2e^{-x} &= 2\cosh(x) - 2\sinh(x) \\
\therefore 9e^x + 2e^{-x} &= 11\cosh(x) + 7\sinh(x)
\end{aligned}$$