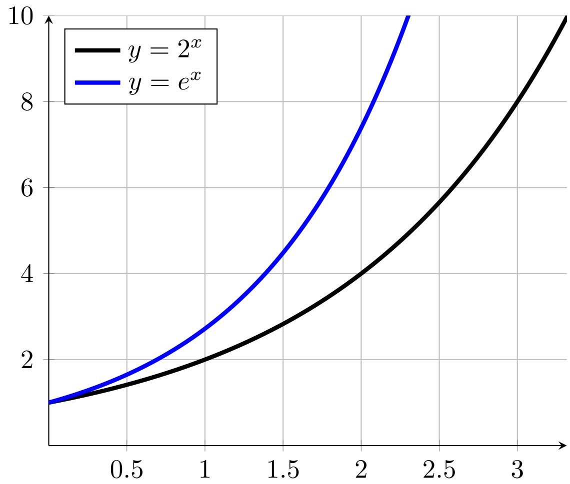
Exponentials And Logarithms

Exponential Functions

For any number a which satisfies $a > 0$, an exponential function is $y = a^x$.

Euler's number $e \approx 2.718$ is of special interest when dealing with exponentials, and the function $y = e^x$ is sometimes written as $exp(e)$. This function is called the exponential function.

Exponential functions often appear in physical problems where the rate of increase or decrease of a quantity is dependent on the value of the quantity at the time. A classic example of this is radioactive decay, where there is an exponential decrease with time.



Logarithms

The logarithm of a particular exponent can be thought of as "undoing" the exponential. For some a and b where $a > 0$ and $b > 0$, $a^x = b$ if and only if $\log_a(b) = x$. This number x is called the logarithm of b to base a.

$y = e^x$ is the inverse of $\ln y = x$. Thus, if $y = 5e^{4x-1}$, $x = 0.25(\ln(0.2y) + 1)$.

$$\begin{aligned} y = 2^x \leftrightarrow && \log_2 y = x \\ y = 3^x \leftrightarrow && \log_3 y = x \\ y = 10^x \leftrightarrow && \log_{10} y = x \\ && \log y = x \\ y = e^x \leftrightarrow && \log_e y = x \\ && \ln y = x \end{aligned}$$

Laws of Logs

The **Laws of Logs** can be used to manipulate equations containing logarithms.

$$\begin{aligned} \log(AB) &= \log(A) + \log(B) \\ \log(\frac{A}{B}) &= \log(A) - \log(B) \\ \log_A(x^B) &= B \log_A(x) \end{aligned}$$

$$\begin{aligned} \log_A(1) &= 0 \\ \log_A(A) &= 1 \\ \log_A(A^B) &= B \end{aligned}$$

Transposing Exponentials and Logs

Given the equation above - $y = 5e^{4x-1}$ - transpose to make x the subject.

1. Divide both sides by 5.
2. Take the natural log (ln) of both sides.
3. Add 1 to both sides.
4. Divide both sides by 4.

$$\begin{aligned} y &= 5e^{4x-1} \\ 0.2y &= e^{4x-1} && (1) \\ \ln(0.2y) &= 4x - 1 && (2) \\ \ln(0.2y) + 1 &= 4x && (3) \\ x &= \frac{\ln(0.2y) + 1}{4} && (4) \end{aligned}$$

Examples

Given $f(t) = 2\exp(-t)$, evaluate f at $t = 0$ and $t = 0.5$.

$$\begin{aligned} f(t) =& 2\exp(-t) \\ =& 2\exp(-0) \\ =& 2 \\ f(t) =& 2\exp(-0.5) \\ =& 1.21 \end{aligned}$$

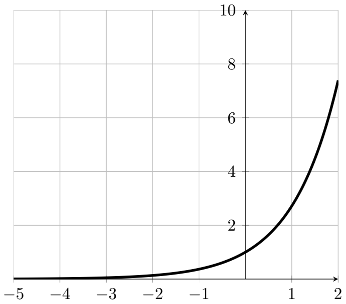
Simplify $(e^x - e^{-x})^2$.

$$\begin{aligned} (e^x - e^{-x})^2 =& (e^x - e^{-x})(e^x - e^{-x}) \\ =& e^{2x} - e^0 - e^0 + e^{2e} \\ =& e^{2x} + e^{-2x} - 2 \end{aligned}$$

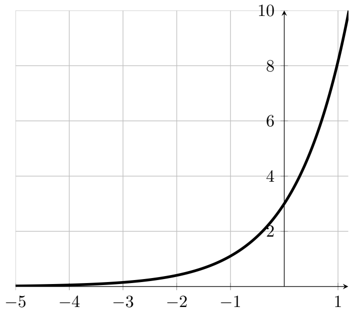
Find the value of x which satisfies $1.76^x = 2002$.

$$\begin{aligned} 1.76^x =& 2002 \\ \log(1.76^x) =& \log(2002) \\ x\log(1.76) =& \log(2002) \\ x =& \frac{\log(2002)}{\log(1.76)} \\ =& 13.45 \end{aligned}$$

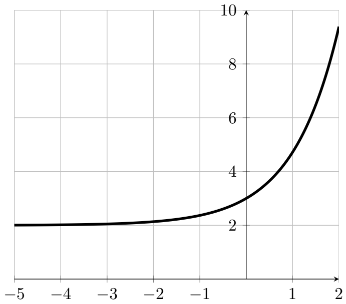
Graphs



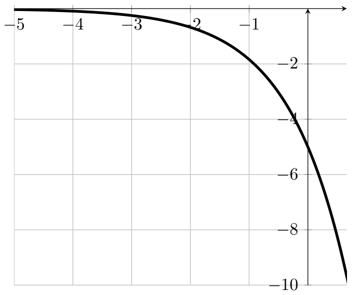
Graph of e^x



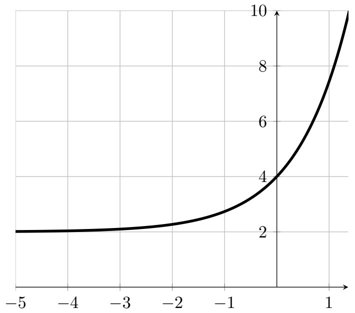
Graph of $y = 3e^x$



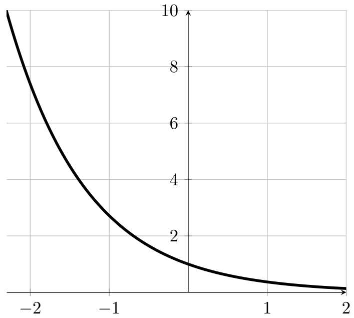
Graph of $y = e^x + 2$



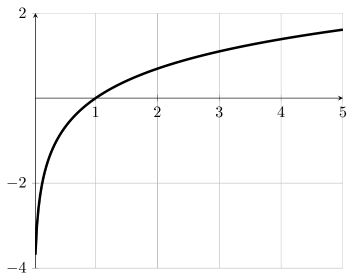
Graph of $y = -5e^x$



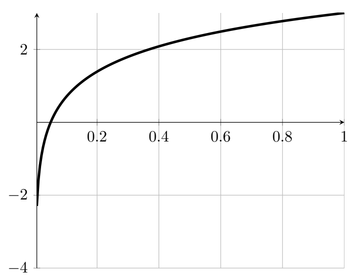
Graph of $y = 2e^x + 2$



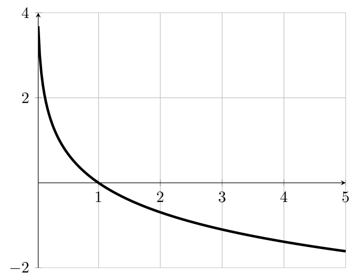
Graph of $y = e^{-x}$



Graph of $y = ln(x)$



Graph of $y = e^x + 2$



Graph of $y = -ln(x)$